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Monte Carlo Simulation of Scattering of Beam Particles and Thermal Photons

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Abstract

Scattering of the stored electrons and positrons with thermal photons is responsible for the majority of single beam particle losses at high energies in LEP. The resulting off-momentum particles constitute a potential source of background to the interaction rate monitors and experiments at LEP. The article describes the Monte Carlo simulation of the process. The program can be easily interfaced to tracking programs to allow studies of off-momentum particle background and single beam lifetime.

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1 Introduction

Scattering of photons on electrons is well known as Compton effect. The special case of scattering of beam particles on thermal photons, radiated off the beam pipe walls, has been observed in LEP and found to be in agreement with theoretical predictions [1]. The quality of the vacuum in LEP is such ($< 10^{-10}$ Torr), that Compton scattering on thermal photons constitutes the dominant contribution to the single beam lifetime at high energies [2]. Due to the high Lorentz-factor at LEP (γ of order 10^5), scattering angles are rather negligible. The energy loss on the electron instead has major consequences. Electrons (and positrons) with 1-2 % energy loss can travel over large distances in LEP. The energy loss in regions with high dispersion generates large betatron oscillations with amplitudes in the order of centimeter. Some fraction of this particles will hit collimators close to the experiments and generate visible background.
A Monte Carlo program has been written to simulate the process in detail.

2 Thermal Photon Spectrum

The spectrum (number of photons with frequency ν to $\nu + d\nu$ in the Volume V) is given by the Planck formula:

$$dn = \frac{8\pi V}{c^3} \frac{\nu^2 \, d\nu}{e^{h\nu/kT} - 1}$$

Substituting $x = h\nu/kT$ and integrating over the whole spectrum we obtain the photon density ρ_γ = n / V

$$\rho_\gamma = 8\pi \left(\frac{kT}{hc}\right)^3 \cdot \int_0^\infty \frac{x^2}{e^x - 1} dx$$

The integral can be expressed using the Riemann-ζ-function :

$$\int_0^\infty \frac{x^2}{e^x - 1} dx = 2 \; \xi(3) \approx 2.404114$$

In 1992, temperature measurements performed at dipoles gave an average value of about $T = 24^\circ C = 295.15 \; K$. The density of photons is $\rho_\gamma = 5.329 \cdot 10^{14} m^{-3}$ and the average photon energy about $2.7 \cdot kT \approx 0.0692 \; eV$. Using the Thomson cross section $\sigma_T = \frac{8\pi}{3} r_e^2 = 0.6652$ barn and the electron charge e, we find for the rate of collisions with thermal photons in LEP (per second, meter, mA of beam current) :

$$\frac{1}{l \cdot i} \cdot \frac{dn}{dt} = \rho_\gamma \cdot \frac{\sigma_T}{e} = 221.0 \; / \; (s \; m \; mA)$$

This number is used as normalization in the Monte Carlo generation. The exact rate for the Compton cross section is a few percent lower. It is determined in the Monte Carlo generation process by comparing the Compton and Thomson cross-section.

2.1 Technique of Monte Carlo Generation of the Thermal Photon Spectrum

The photon spectrum in the laboratory frame is generated at random by subsequent calls in FORTRAN to a function without arguments of the form : " E_γ = kT · PLAGEN() ". To generate efficiently random numbers in the interval $[0, \infty)$ following the probability of the Planck spectrum

$$f(x) = \frac{x^2}{e^x - 1} \quad ,$$

a combination of two standard Monte Carlo generation techniques is used:
i) inverse transform method
ii) rejection
An example of the combination of this methods can also be found in [4]. For method i), we have to find one or several approximations f_i to the function f, that can be integrated

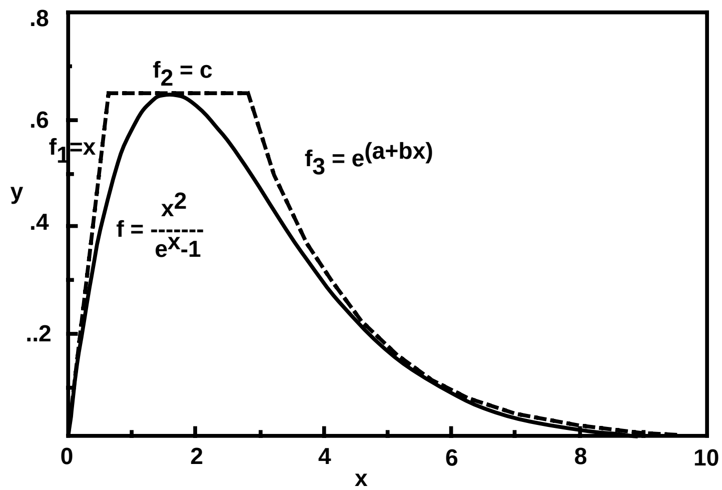

Figure 1: Exact Planck function f and the three approximate functions f_1 to f_3 used internally in the generation of the Planck spectrum

and inverted. Here we use three simple approximations f_1 to f_3 that are listed in table 1 and illustrated in fig. 1. The approximations are chosen such, that the relation $f_i(x) > f(x)$ or weight $= f(x)/f_i(x) < 1$ holds in the interval where the approximation is

i	interval	$f_i(x)$	$F_i(x) = \int f_i(x)dx$	$F_i^{-1}(r)$	mean weight
1	0 - x_1	x	$x^2/2$	$x_1 \cdot \sqrt{r}$	0.8015
2	x_1 - x_2	c	cx	$x_1 + (x_2 - x_1) \cdot r$	0.9165
3	$x_2 - \infty$	exp(a+bx)	exp(a+bx) / b	$x_2 + log(r)/b$	0.9047

Table 1: Approximate functions, their integral and the transformation applied on the random number r (flat in the interval 0 - 1) derived from the inverse of the integral. They are used internally in the generation of the Planck spectrum. The values for the constants are : a=1.266, b=-0.6, c=0.648 ; x_1 = c ; $x_2 = [log(c) - a]/b$

used. The weight is used in method ii) to obtain the exact spectrum with unity weight. The average CPU time per call to PLAGEN is below 4 μsec (CERN central IBM 9000).

3 Compton Scattering of Beam Particles and Thermal Photons

The mean final photon energy can be roughly estimated as follows: We saw that the average thermal photon energy is about $k_i = 0.07\text{eV} = 7 \cdot 10^{-11}\text{GeV}$. For a beam energy of 50 GeV we have $\gamma \approx 10^5$. The angle between the incoming electron and photon is called ψ. From Lorentz transformation we obtain for the photon energy in the electron rest frame

$$k^* = \gamma\ k_i\ (1 - \beta \cos \psi)$$

or in average approximately $k^* = k_i \cdot \gamma = 7 \cdot 10^{-6}$ GeV. This is small compared to the electron mass and the Compton scattering is well approximated by the elastic Thomson scattering process. The scattered photon energy is $k^{*'} \approx k^*$. The angular distribution of the Thomson scattering is $1 + \cos^2 \theta$. In average, another factor of γ is gained in the Lorentz-transformation back into the laboratory system such that the final photon energy becomes

$$k' \approx \gamma \cdot k^* \approx \gamma^2 \cdot k_i \quad \approx 0.7 \text{ GeV} \quad \text{for } \mathrm{E}_b = 50 \text{ GeV} .$$

The mean photon energy is effectively boosted by γ^2. From the detailed simulation we obtain an average scattered photon energy at LEPI (45.6 GeV) of 0.52 GeV or 1.1 % of the beam energy and 1.9 GeV or 2.2 % for LEP II with 90 GeV beam energy.
From the Lorentz boost after the Compton scattering, the angle of the photon with respect to the beam will be typically $1/\gamma$. Multiplying this with the ratio of electron to photon energy, we obtain electron scattering angles in the laboratory frame of $\mathcal{O}\,(10^{-7})$ or $\mathcal{O}\,(0.1\,\mu\text{rad})$, independent of beam energy.

3.1 Monte Carlo Simulation of the Compton Scattering

The simulation program has been written in a general and modular way, using subroutines for the lorentz-boost and rotation. Explicit formulas for a special choice of coordinates can be found in [3].
The simulation proceeds in a number of steps:

- $i)$ We start with an incoming beam particle (electron or positron) and a thermal photon with random direction (isotropy) in the laboratory system.
- $ii)$ Lorentz-transformation of the photon into the electron rest frame.
- $iii)$ Rotate photon to +z direction.
- $iv)$ Generation of Compton scattering angles using the Thomson cross section as approximation.
- $v)$ Rejection of a few % of events to correct for the ratio Compton / Thomson

cross section.

$vi)$ Rotate the scattered photon (using the inverse matrix of step iii).

$vii)$ Lorentz transform the photon back into the LAB frame. The scattered electron coordinates are obtained from energy-momentum conservation.

The generation of the process is done in steps iv) and v), the rest is standard coordinate transformations. The differential Thomson cross section

$$\frac{d\sigma_T}{d\Omega} = \frac{r_e^2}{2} \cdot (1 + \cos^2\theta)$$

can be directly generated using a set of random numbers r_i, uniformly distributed in the interval (0,1). The ϕ angle is obtained using $\phi = 2\pi \cdot r_1$.
The maximum value of n random numbers has the probability distribution $f(x) = x^{n-1}$ [5]. This is used to generate θ following a $1 + \cos^2\theta$ distribution by choosing a flat

$$\cos\theta = 2 \cdot r_2 - 1$$

distribution in 75 % of the cases and

$$\cos\theta = \pm\text{max}(r_3, r_4, r_5)$$

for the remaining 25 %.
The Compton cross-section can be written as

$$\frac{d\sigma}{d\Omega} = \frac{r_e^2}{2} \cdot \left[1 + \cos^2\theta + x + 1/x - 2\right] \cdot x^2$$

x is the ratio of the scattered to the unscattered photon energy

$$x = \frac{k'}{k} = \left[1 + \frac{k}{m_e} \cdot (1 - \cos\theta)\right]^{-1}$$

The maximum value of x=1 is reached for forward scattering. The minimum value is:

$$x = (1 + 4\gamma k^*/m_e)^{-1} \approx 1 - 4\gamma k^*/m_e \quad \approx .95 \text{ for LEPI and } 0.90 \text{ for LEPII}$$

The case x=1 represents the elastic limit where Compton and Thomson cross section are equal. The ratio Compton cross section divided by the Thomson cross section can be regarded as weight and is shown in figure 2. The exact spectrum with unity weight is obtained using the rejection technique. The effective Compton cross section σ_T for scattering with thermal photons is calculated from the Thomson cross section and the ratio of accepted / total events (equal to the mean weight) :

$$\text{for } E_b = 45.6 \text{ GeV} \quad : \quad w_{mean} = 0.9767 \quad \sigma_C = \sigma_T \cdot w_{mean} = 0.6497 \text{ barn}$$
$$\text{for } E_b = 90 \text{ GeV} \quad : \quad w_{mean} = 0.9558 \quad \sigma_C = \sigma_T \cdot w_{mean} = 0.6358 \text{ barn}$$

The energy spectrum of the scattered photons, normalized to the beam energy, is shown in figure 3. The CPU consumption per event (Planck + Compton) is about 25 μsec (CERN central IBM 9000).

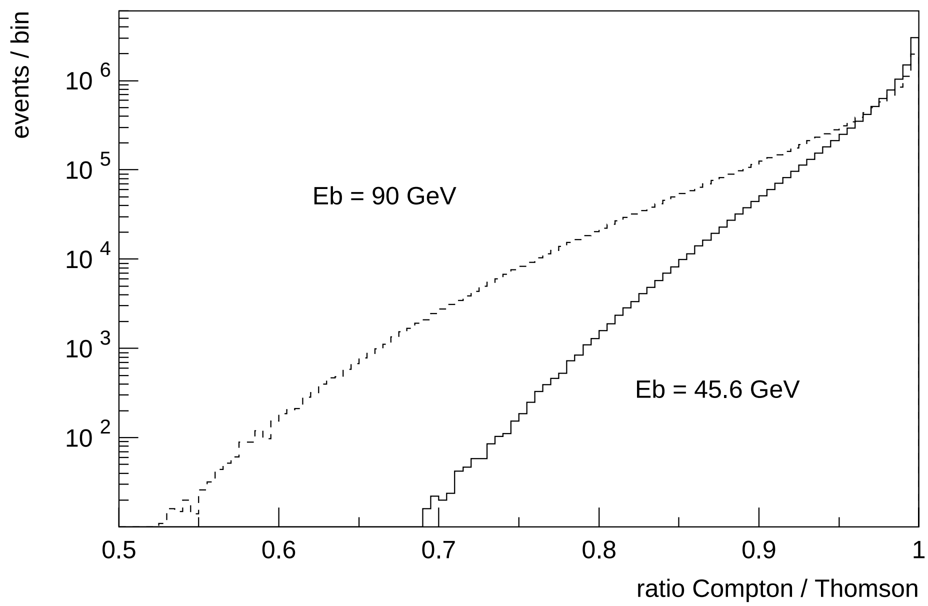

Figure 2: Ratio of Compton / Thomson cross sections for scattering of beam particles with thermal photons.

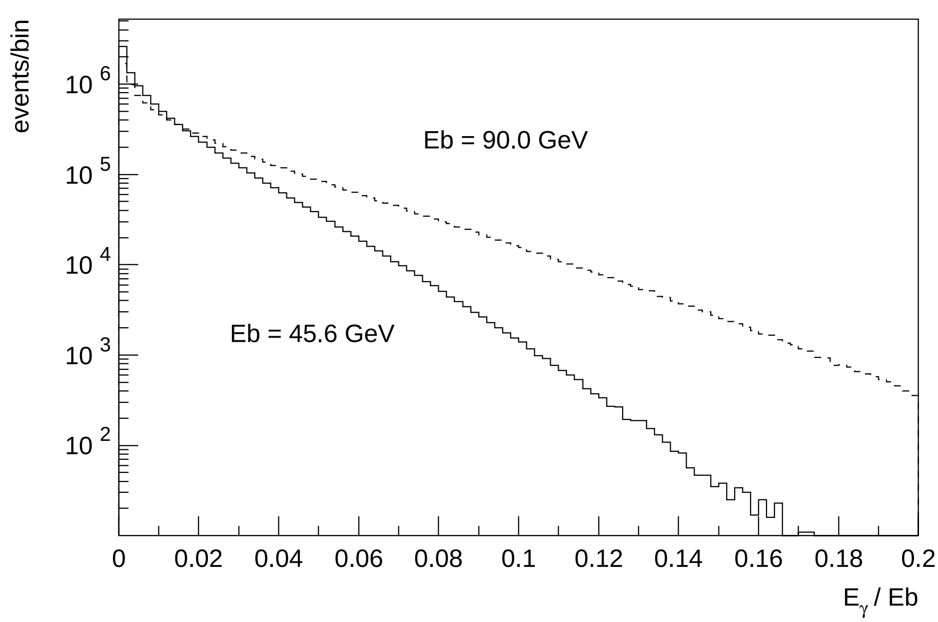

Figure 3: Energy spectrum of the scattered photons, normalized to beam energy for LEP I and LEP II energies.

3.2 Interfacing with Tracking Programs

The FORTRAN code for thermal photon scattering is called THEGEN FORTRAN and can be obtained from the author (HBU at CERNVM). It also contains the FUNCTION PLAGEN to generate the Planck spectrum. The interfacing with tracking programs is done by means of four vectors, stored in a COMMON block. The incoming beam particle four vector is called QP. The first three components are the momenta px, py, pz and the fourth component is the energy. The outgoing beam particle four vector is called QOUT and the photon vector QK. All four momentum numbers are in GeV. The two integers ITHGEN, ITHACC are also stored in the same common and represent the number of trials and accepted events in the generation. The calling program will have to include the following steps:

Initialization :

- Calculation of photon density and
- approximate scattering probability using the Thomson cross section σ_T

Generation :

- fill four vector QP (incoming beam particle)
- E_γ = kT · PLAGEN()
- CALL THEGEN(E_γ)
- QOUT, QK now represent the outgoing beam particle and photon momenta

The corrected cross section is obtained as obtained as σ_T · ITHACC / ITHGEN

4 Beam Lifetime

The probability for a single electron or positron to interact with a thermal photon per unit length is given by the product of photon density and effective Compton cross section $\rho_\gamma \cdot \sigma_C$. Multiplying with the speed of light we obtain the probability per unit time. If for a fraction f of the collisions, the beam particle is lost, we obtain for the normalized loss rate or inverse lifetime :

$$\frac{1}{\tau} = \rho_\gamma \cdot c \cdot f \cdot \sigma_C$$

Using the Thomson cross section and f=1 we obtain the lower limit on the lifetime from scattering of beam particles with thermal photons (for $24^\circ C$) :

$$\tau_{min} = \frac{1}{\rho_\gamma \; c \; \sigma_T} = 26.17 \text{ hours}$$

The lifetime as function of the energy acceptance is given in table 2. It is interesting to compare this with the observed lifetimes in polarization runs given in table 3. The calculation was done assuming maximal aperture (only limited by the bucket height). The real aperture with non-linear fields and collimators will be somewhat smaller and can be studied using tracking programs interfaced to the thermal photon scattering simulation. Still, we account already for the majority of the single beam losses by scattering with thermal photons. Other losses (reduced aperture, beam-gas etc.) contribute only with partial lifetimes of about 100 hours or more.

$\Delta E/E$	for E_b = 45.6 GeV		for E_b = 90.0 GeV	
%	f	τ [h]	f	τ [h]
.2	.7375	36.33	.8311	32.94
.4	.6028	44.44	.7329	37.35
.6	.5067	52.87	.6577	41.63
.8	.4322	61.98	.5959	45.94
1.0	.3722	71.97	.5435	50.37
1.2	.3225	83.07	.4980	54.97
1.4	.2808	95.41	.4580	59.77
1.6	.2453	109.22	.4223	64.82
1.8	.2150	124.63	.3904	70.13
2.0	.1886	142.03	.3615	75.73

Table 2: Fraction f of collisions with thermal photons leading to particle loss and the resulting lifetime as function of the energy acceptance $\Delta E/E$ for LEP I and LEP II energies

Optics	Q_s	momentum compaction	bucket height %	τ seen [h]	τ thermal [h]
60/60°	0.085	$3.867 \cdot 10^{-4}$	0.78	39 ± 1	61.2
90/60°	0.0625	$1.859 \cdot 10^{-4}$	1.28	48 ± 2	87.5

Table 3: Comparison of observed single beam lifetimes and the calculated single lifetime contribution from thermal photon scattering. The measured values are obtained from polarization runs with the 60/60° and 90/60° optics. The synchrobetatron tune Q_s, momentum compaction factor and the calculated bucket (half) height used as energy cutoff in the calculation are also given.

References

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